EFFECT OF INITIAL PARTICLE INSTABILITY IN MUON COLLISIONS

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Abstract

We study $\mu^+\mu^-$ collision with production of some state F (for example, $F = W^+$) and $(e\bar{\nu})$ system with the effective mass, which is less than muon mass. In this case the momentum k, transferred from μ to $e\bar{\nu}$ system (virtual neutrino momentum), can be time–like. The path of integration over k^2 goes through the pole at $k^2 = 0$. It gives divergent cross section.

This divergence disappears if the finite width Γ of initial μ is taken into account. It gives the factor $\propto \Gamma^{-1}$ in cross section. The compensating factor Γ appears from $\mu \to e\bar{\nu}\nu$ subprocess. The resulting cross section is finite but high enough.

The modern discussions about muon colliders (see e.g. [1]) provide new place for studying of effects related to particle instability.

For the definiteness, we consider the process (momenta of particles are shown in brackets):

$$\mu^-(p_1)\mu^+(p_2) \to e(q_1)\bar{\nu}(q_2)W^+(p_3). \tag{1}$$

We use the following notations: M is the W boson mass, m is the muon mass, $s \equiv 4E^2 = (p_1 + p_2)^2$; $x = M^2/s$; $q = q_1 + q_2$, $k = p_1 - q \equiv p_3 - p_2$; we neglect electron mass.

1. We study the specific kinematical region where the effective mass of $e\bar{\nu}$ system is less than muon mass, $q^2 < m^2$. In this case the transferred momentum k can be time–like, its maximal value is positive:

$$k^2 < t_{max} = \frac{x}{1-x}\left[m^2(1-x) - q^2\right] > 0. \tag{2}$$

With the growth of total transverse momentum of produced system, this transferred momentum becomes space–like, $k^2 < 0$. Therefore the integration over this transverse momentum (at fixed q) goes through the point $k^2 = 0$.

The main contribution to the cross section in this region is given by the diagram with neutrino exchange in t–channel, this (virtual) neutrino momentum is k. This

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contribution gives a factor $(k^2)^{-2}$ in the matrix element squared. The standard integration over k^2 results in divergent cross section in this case[1]!

This paradox originates from the instability of muon, decaying into $e\bar{\nu}\nu$ system. The point $k^2 = 0,\ q^2 < m^2$ is within the physical region for this decay. (The obtained result is conserved if neutrino have some small mass. Since muon decays into $e\nu\bar{\nu}$, the integration over k^2 goes through the pole of neutrino propagator independent of value of neutrino mass.)

2. This divergence is killed, if one takes into account the fact that the wave function differs from the standard plane wave due to muon instability. In muon rest frame:

$$e^{-imt/\hbar} \Rightarrow e^{-i(m-i\Gamma/2)t/\hbar}. \tag{3}$$

In this frame the 4-momentum of μ^- is $\tilde{p}_1 = (m - i\Gamma/2, 0, 0, 0)$. To obtain the energy of produced $e\bar{\nu}$ system $\tilde{q}^0$ in this system, we use simple kinematical relations $2p_1q = m^2 + q^2 - k^2,\quad 2p_1q \equiv 2m\tilde{q}^0$. It results in $\tilde{q}^0 = (m^2 + q^2 - k^2)/2m$.

New value of k^2 is obtained from relation $k^2_{new} \equiv (p_1 - q)^2 = m^2 - im\Gamma + q^2 + i\Gamma\tilde{q}^0$. Using the above value of $\tilde{q}^0$, we obtain with the necessary accuracy[2]:

$$k^2 \Rightarrow k^2 - i\gamma; \quad \gamma = \frac{m\Gamma(m^2 - q^2)}{2m^2}. \tag{4}$$

Now we calculate the cross section of the process in the standard way. Calculations are simplified, since we neglect small quantities $\sim m^2/M^2$, Γ/m in the result. Finally we get:

$$\begin{aligned} d\sigma &= \frac{|\mathcal{M}|^2 dk^2 dq^2}{4(4\pi)^3 s(s - 4m^2)} \frac{d\Omega^*_q}{4\pi} \frac{d\varphi}{2\pi}; \\ |\mathcal{M}|^2 &= \frac{(4\pi\alpha)^3}{M^2(\sin^2\Theta_W)^3} \frac{(2q_1p_1)(2kq_2)}{k^4 + \gamma^2}. \end{aligned} \tag{5}$$

Here $d\Omega^*_q$ is the solid angle element in the center of mass of produced $e\bar{\nu}$ system.

The subsequent angular integration is trivial. The integration over k^2 gives π/γ for any q^2, since the bounds of the integration region are much higher than γ. The integration over q^2 covers the region $q^2 < m^2(1 - x)$, where exchanged neutrino can be on mass shell (2). This procedure is very close to the calculation of the muon decay width. We insert this width instead of the corresponding combination of factors to the eq. (5). It compensates the factor Γ in the denominator, and the final result is:

$$\sigma = \frac{\pi^2\alpha}{s\sin^2\Theta_W} f(x) \equiv 20x f(x) \text{ nb}; \tag{6}$$

$$f(x) = 4x(1 - x)(2 - x). \tag{7}$$

This equation gives us the solution of discussed problem. With above prescription we obtain the finite cross section.

Let us discuss briefly some ideas about the interpretation of this solution.

First, it seems to be natural to consider this phenomenon as an effect of a real μ decay. The obtained quantities correspond to the integration over whole space–time.

[1] I was informed about this paradox by F. Boudjema.

[2] This way for derivation of k^2_{new} was proposed by V.G. Serbo.

This interpretation is supported by the fact, that the total number of neutrino is the same as the number of muons, $\int f(x)dx = 1$.

The other idea is to consider this effect as some "induced" decay of muon in the field of spectator. If this approach is correct, then these muon colliders can be also used as $\mu\nu$, $\nu\nu$,... colliders with high enough luminosity.

To finally establish which interpretation is really correct, more detail calculations taking into account sizes of beam and interaction region are needed.

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References

[1] R.B. Palmer. Beam Dynamics Newsletter. # 8 (1995).

Appendix

If the second interpretation is realized, we have very promising picture.

First of all, the quantity $f(x)$ (7) can be considered as the neutrino component of the muon. Indeed, our cross section can be written through the "number of equivalent neutrino" $F(x)dx$ with the energy in the interval $\{xE; (x+dx)E\}$ as $\int F(x)dx \cdot \sigma_{\mu\nu\to W}(xs)$. The standard approximation for the cross section of the subprocess $\mu\nu \to W$ with unpolarized muons is (S is spin of W boson):

$$\sigma_{\mu\nu\to W}(\hat{s}) = (2S+1)4\pi^2 \frac{\Gamma_{W\to\mu\nu}}{M} \delta(\hat{s} - M^2); \quad \Gamma_{W\to\mu\nu} = \frac{\alpha}{12\sin^2\Theta_W}.$$

With these equations, we obtain from eq. (6) $F(x) \equiv f(x)$, in other words $f(x)dx$ is the "number of equivalent neutrino" with the energy in the above interval. (This result was reproduced by V.G. Serbo by the method, similar to that for the derivation of equivalent electron method).

The above calculations determine the "flux of equivalent muonic neutrino". The same approach give fluxes of electrons and electronic neutrino in muon.

The calculation repeats the above one completely. The effective μ decay vertex is reduced, in fact, (with the accuracy m^2/M^2) to the well known form of four–fermion interaction which admits Pauli–Fierz transformation with exchange electronic and muonic neutrino:

$$\left(\hat{\mu}^- \gamma^\alpha (1-\gamma^5) \hat{\bar{\nu}}_\mu\right) \left(\hat{e}^+ \gamma^\alpha (1-\gamma^5) \hat{\nu}_e\right) = \left(\hat{\mu}^- \gamma^\alpha (1-\gamma^5) \hat{\nu}_e\right) \left(\hat{e}^+ \gamma^\alpha (1-\gamma^5) \hat{\bar{\nu}}_\mu\right)$$

Therefore, the flux of electronic neutrino in muon coincides with that for muonic neutrino (7). The same is true for the electron flux.

For the given mass of produced system, the energy dependence of the cross section (6) is determined by the quantity $xf(x)$. Its highest value is 0.907 at $x = (9-\sqrt{17})/8 = 0.61$. This quantity is larger than one half of this highest value for x within interval $\{0.285; 0.88\}$. Therefore, high cross section for the production of some particle with mass $\tilde{M}$ (W boson or charged Higgs or heavy new W boson, etc.) in the $\nu\mu$ collision is achieved at $\sqrt{s} = 1.06 \div 1.87\tilde{M}$. In particular, the of cross section shows that $\mu^+\mu^-$ collider with $\sqrt{s} = 130$ GeV and luminosity about 10^{31} cm$^{-2}s^{-1}$ will be the W–boson factory. The number of produced W's will be here approximately the same as the number of Z–bosons at LEP.

In this case, the $\mu\mu$ colliders will be simultaneously $\mu\nu_\mu$, $\mu\nu_e$, μe, $e\nu_\mu$, $\nu\nu$ colliders with the same luminosity as in the basic $\mu\mu$ collisions (but with nonmonochromatic new beams).